A DIFFERENT VIEW ON THE VIRTUAL TRANSITION. WHEN IS A TRANSITION CURVE NOT NEEDED?

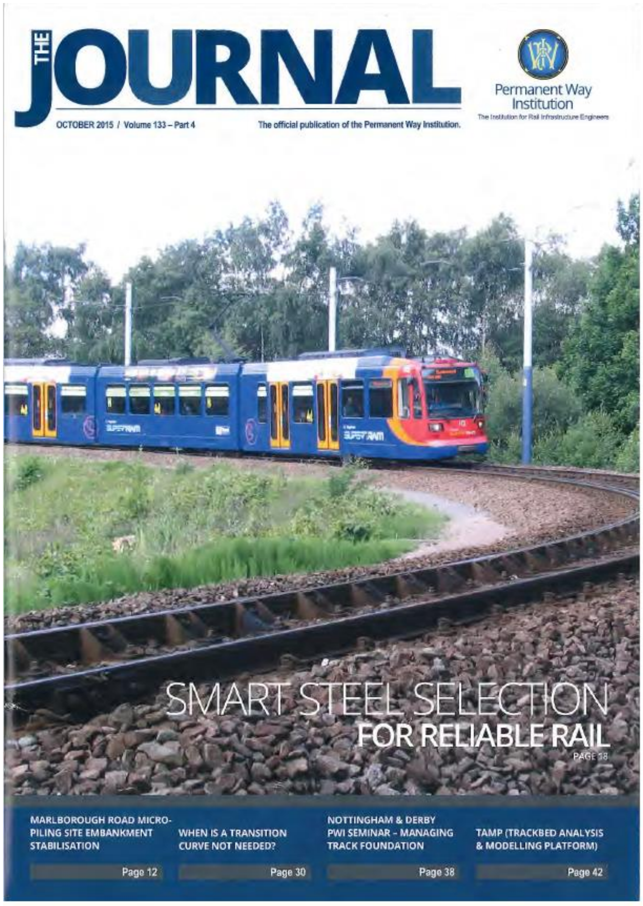
by Constantin Ciobanu

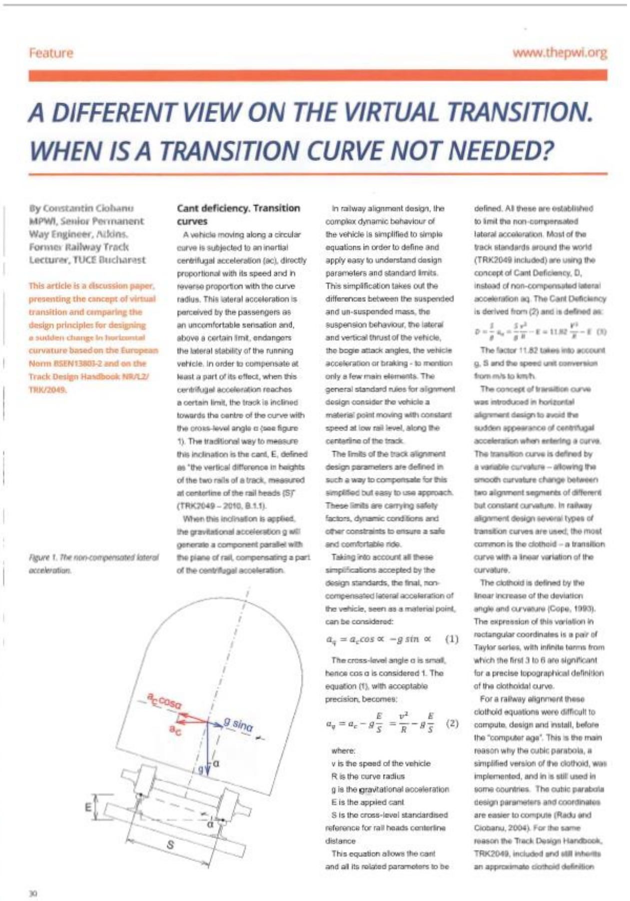
constantin.ciobanu@atkinsglobal.com

MPWI, MCIHT, Senior Permanent Way Engineer, Atkins

former Railway Track Lecturer, TUCE Bucharest

This article was published in the Permanent Way Institution Journal, October 2015 / Volume 13 – Part 4, pages 30-36





Feature | www.thepwi.org

A DIFFERENT VIEW ON THE VIRTUAL TRANSITION. WHEN IS A TRANSITION CURVE NOT NEEDED?

By Constantin Ciobanu
MPWI, Senior Permanent Way Engineer, Atkins.
Former Railway Track Lecturer, TUCE Bucharest

This article is a discussion paper, presenting the concept of virtual transition and comparing the design principles for designing a sudden change in horizontal curvature based on the European Norm BSEN13803-2 and on the Track Design Handbook NR/L2/TRK/2049.

Cant deficiency. Transition curves

A vehicle moving along a circular curve is subjected to an inertial centrifugal acceleration (ac), directly proportional with its speed and in reverse proportion with the curve radius. This lateral acceleration is perceived by the passengers as an uncomfortable sensation and, above a certain limit, endangers the lateral stability of the running vehicle. In order to compensate at least a part of its effect, when this centrifugal acceleration reaches a certain limit, the track is inclined towards the centre of the curve with the cross-level angle α (see figure 1). The traditional way to measure this inclination is the cant, E, defined as "the vertical difference in heights of the two rails of a track, measured at centerline of the rail heads (S)" (TRK2049 – 2010, B.1.1).

When this inclination is applied, the gravitational acceleration g will generate a component parallel with the plane of rail, compensating a part of the centrifugal acceleration.

Figure 1. The non-compensated lateral acceleration.

In railway alignment design, the complex dynamic behaviour of the vehicle is simplified to simple equations in order to define and apply easy to understand design parameters and standard limits. This simplification takes out the differences between the suspended and un-suspended mass, the suspension behaviour, the lateral and vertical thrust of the vehicle, the bogie attack angles, the vehicle acceleration or braking - to mention only a few main elements. The general standard rules for alignment design consider the vehicle a material point moving with constant speed at low rail level, along the centerline of the track.

The limits of the track alignment design parameters are defined in such a way to compensate for this simplified but easy to use approach. These limits are carrying safety factors, dynamic conditions and other constraints to ensure a safe and comfortable ride.

Taking into account all these simplifications accepted by the design standards, the final, non-compensated lateral acceleration of the vehicle, seen as a material point, can be considered:

$$a_q = a_c \cos \alpha - g \sin \alpha \quad (1)$$

The cross-level angle α is small, hence cos α is considered 1. The equation (1), with acceptable precision, becomes:

$$a_q = a_c - g\frac{E}{S} = \frac{v^2}{R} - g\frac{E}{S} \quad (2)$$

where:
v is the speed of the vehicle
R is the curve radius
g is the gravitational acceleration
E is the applied cant
S is the cross-level standardised reference for rail heads centerline distance

This equation allows the cant and all its related parameters to be defined. All these are established to limit the non-compensated lateral acceleration. Most of the track standards around the world (TRK2049 included) are using the concept of Cant Deficiency, D, instead of non-compensated lateral acceleration aq. The Cant Deficiency is derived from (2) and is defined as:

$$D = \frac{S}{g} a_q = \frac{S}{g}\frac{v^2}{R} - E = 11.82\frac{V^2}{R} - E \quad (3)$$

The factor 11.82 takes into account g, S and the speed unit conversion from m/s to km/h.

The concept of transition curve was introduced in horizontal alignment design to avoid the sudden appearance of centrifugal acceleration when entering a curve. The transition curve is defined by a variable curvature – allowing the smooth curvature change between two alignment segments of different but constant curvature. In railway alignment design several types of transition curves are used; the most common is the clothoid – a transition curve with a linear variation of the curvature.

The clothoid is defined by the linear increase of the deviation angle and curvature (Cope, 1993). The expression of this variation in rectangular coordinates is a pair of Taylor series, with infinite terms from which the first 3 to 6 are significant for a precise topographical definition of the clothoidal curve.

For a railway alignment these clothoid equations were difficult to compute, design and install, before the "computer age". This is the main reason why the cubic parabola, a simplified version of the clothoid, was implemented, and in is still used in some countries. The cubic parabola design parameters and coordinates are easier to compute (Radu and Ciobanu, 2004). For the same reason the Track Design Handbook, TRK2049, included and still inherits an approximate clothoid definition

30

A DIFFERENT VIEW ON THE VIRTUAL TRANSITION. WHEN IS A TRANSITION CURVE NOT NEEDED?

by Constantin Ciobanu
constantin.ciobanu@atkinsglobal.com
MPWI, MCIHT, Senior Permanent Way Engineer, Atkins
former Railway Track Lecturer, TUCE Bucharest

Abstract

This article is envisaged as a discussion paper, presenting the concept of virtual transition and comparing the design principles for designing a sudden change in horizontal curvature based on the European Norm BSEN13803-2 and on the Track Design Handbook NR/L2/TRK/2049. It introduces various track engineering data, suggesting an informative answer to the question: ***when is a transition curve not needed?***

Cant deficiency. Transition curves

A vehicle moving along a circular curve is subjected to an inertial centrifugal acceleration (a_c), directly proportional with its speed and in reverse proportion with the curve radius. This lateral acceleration is perceived by the passengers as an uncomfortable sensation and, above a certain limit, endangers the lateral stability of the running vehicle. In order to compensate at least a part of its effect, when this centrifugal acceleration reaches a certain limit, the track is inclined towards the centre of the curve with the cross-level angle α (see figure 1). The traditional way to measure this inclination is the cant, E, defined as *"the vertical difference in heights of the two rails of a track, measured at centerline of the rail heads (S)"* (TRK2049 – 2010, B.1.1).

When this inclination is applied, the gravitational acceleration g will generate a component parallel with the plane of rail, compensating a part of the centrifugal acceleration.

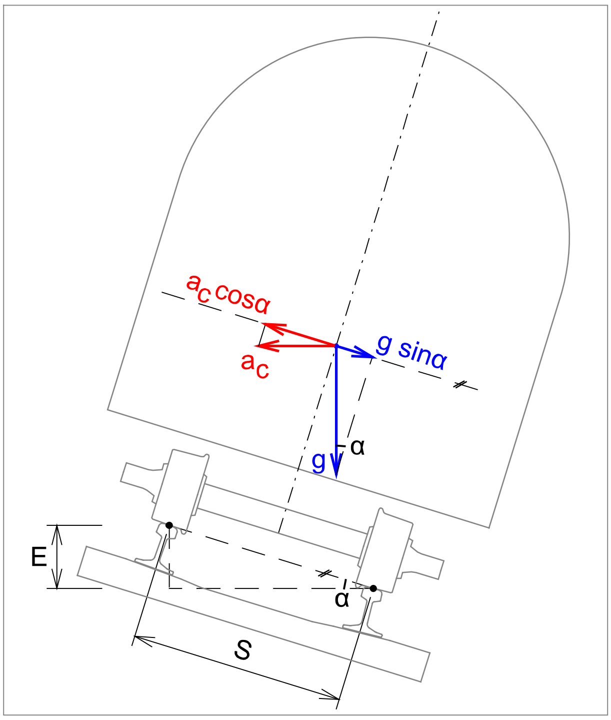


Figure 1. The non-compensated lateral acceleration

In railway alignment design the complex dynamic behaviour of the vehicle is simplified to simple equations in order to define easy to understand and apply design parameters and standard limits. This simplification takes out the differences between the suspended and un-suspended mass, the suspension behaviour, the lateral and vertical thrust of the vehicle, the bogie attack angles, the vehicle acceleration or braking - to mention only a few main elements. The general standard rules for alignment design consider the vehicle a material point moving with constant speed at low rail level, along the centerline of the track.
The limits of the track alignment design parameters are in such a way defined to compensate for this simplified but easy to use approach. These limits are carrying safety factors, dynamic conditions and other constraints to ensure a safe and comfortable riding.

Taking into account all these simplifications accepted by the design standards, the final, non-compensated lateral acceleration of the vehicle, seen as a material point, can be considered:

$$a_q = a_c cos \propto - g\ sin\ \propto \quad (1)$$

The cross-level angle α is small, hence cos α is considered 1. The equation (1), with acceptable precision, becomes:

$$a_q = a_c - g\frac{E}{S} = \frac{v^2}{R} - g\frac{E}{S} \quad (2)$$

where:

v is the speed of the vehicle
R is the curve radius
g is the gravitational acceleration
E is the applied cant
S is the cross-level standardised reference for rail heads centerline distance

This equation allows to define the cant and all its related parameters. All these are established to limit the non-compensated lateral acceleration. Most of the track standards around the world (TRK2049 included) are using the concept of Cant Deficiency, D, instead of non-compensated lateral acceleration a_q. The Cant Deficiency is derived from (2) and defined as:

$$D = \frac{S}{g}\,a_q = \frac{S}{g}\frac{v^2}{R} - E = 11.82\,\frac{V^2}{R} - E \quad (3)$$

The factor 11.82 takes into account g, S and the speed unit conversion from m/s to km/h.

The concept of transition curve was introduced in the horizontal alignment design to avoid the sudden appearance of the centrifugal acceleration when entering a curve. The transition curve is defined by a variable curvature – allowing the smooth curvature change between two alignment segments of different but constant curvature.
In the railway alignment design several types of transition curves are used; the most common is the clothoid – a transition curve with a linear variation of the curvature.
The clothoid is defined by the linear increase of the deviation angle and curvature (Cope, 1993). The expression of this variation in rectangular coordinates is a pair of Taylor series, with infinite terms from which the first 3 to 6 are significant for a precise topographical definition of the clothoidal curve.
For a railway alignment these clothoid equations were difficult to compute, design and install, before the “computer age”. This is the main reason why the cubic parabola, a simplified version of the clothoid, was implemented, and in some countries still used. The cubic parabola design parameters and coordinates are easier to compute (Radu and Ciobanu, 2004). For the same reason the Track Design Handbook, TRK2049, included and still inherits an approximate clothoid definition with only two terms for x and y coordinates.
The complexity of clothoid definition was overcome with the introduction of the computer aided design and the appearance of specialised design software. To design a clothoid nowadays the designer needs to define only the length of the transition and the start and ending radius. Based on these data the design software can easily compute all the intrinsic parameters and coordinates of the clothoid.
The first track machine on-board computer was introduced in 1976 by Plasser&Theurer but the real computer guided maintenance machine appeared after the implementation of ALC software on Plasser&Theurer machines in 1989 (Plasser&Theurer, 2015). In time, the software and the technologies have improved, allowing the design, installation and maintenance of complex track geometry, providing

precise coordinates even of non-linear transition curves. The track installation and maintenance machines currently in use are able to ensure an increased precision of installation and mechanised maintenance. This allows the transition curves to have a similar maintainability as a circular curve.

Track geometry standard deviation (SD) in the area of curvature change

A good way to evaluate the difference between an alignment with or without a transition curve is to estimate the track geometry standard deviation (SD) for a perfectly installed alignment. This can be done using the Network Rail recommended software, the Track Geometry Standard Deviation Calculator (Network Rail, 2008).
Figure 2 presents the design track geometry standard deviations for a simple horizontal alignment, a straight followed by a circular curve, with no transition curve or with transition length of 30, 40 respectively 50m. For all these cases the cant is zero.
For the 35m Horizontal Alignment SD filter, comparing the estimated SD for the two types of alignments, with or without transition between the straight element and the circular curve, we find an average SD **2.3** times smaller for a 30m transition alignment compared to the alignment without transition curve.

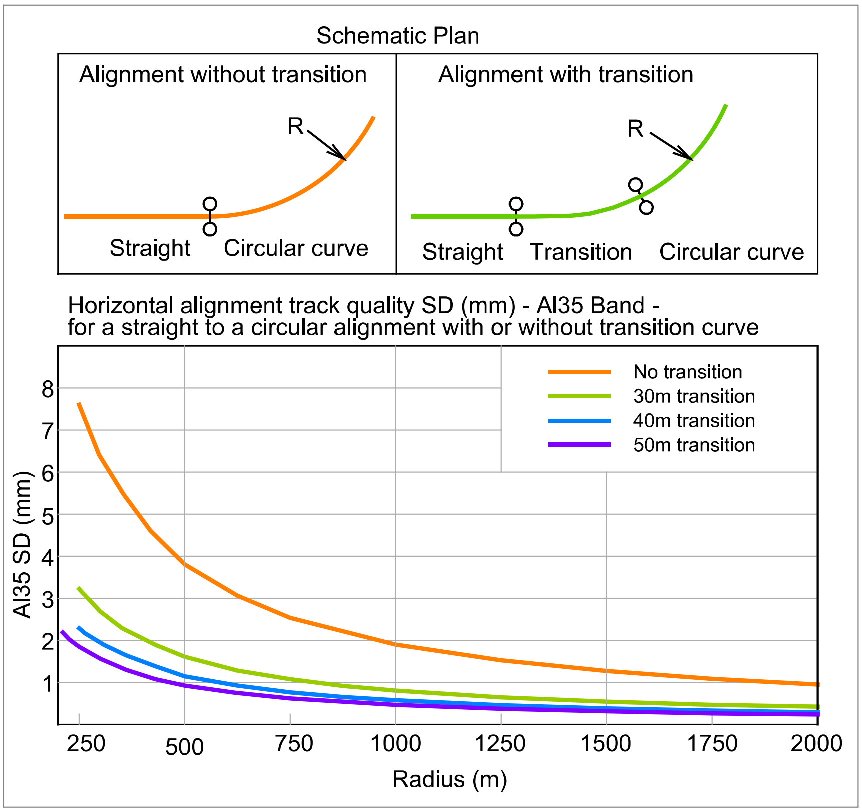


Figure 2. Horizontal alignment track quality standard deviation - SD (mm) - Al35 Band for a straight to a circular alignment with or without transition curve

Similar results can be found for the other alignment combinations – successive circular curves of similar or reverse direction of curvature.
One very important fact to be noted here is that ***the transition curve is causing horizontal alignment standard deviation (SD) even for a perfectly installed geometry,*** in other words the transition curve has an inherent standard deviation. This is due to the curvature variation along the transition, naturally causing oscillations of the traveling vehicle and hence recorded by the Track Recording Vehicle. This is sometimes misinterpreted as an inability of the maintenance machines to install or maintain a rigorously defined transition – which is not true. The installation and maintenance tolerances of the transition curves are similar to the ones for a straight or a curved track. This inherent track quality standard deviation will appear even for a perfect installed transition. The only way this standard deviation can be improved (decreased) is either by increasing the length of the transition or, if possible, by using an improved transition geometry - a classic non-linear transition for example.

On the other hand, ***the sudden change in curvature – the absence of transition – will cause significantly higher SD than the one found when the transition curve is used***, even for a perfectly installed alignment. In the same way, during track geometry measurements, the Track Recording Vehicle will record higher SD in this case compared to the one for a transition curve.

In the case of the actual installation, the sudden change in curvature is practically impossible to be installed on track, as the rails are not kept in place laterally by a perfectly rigid system, especially for a ballasted track. In the area of a designed sudden change in curvature there will always be a short segment of variable curvature, an *"installation transition"*, not found in the design. This will happen because the complex track system will elastically bend to smoothen the sudden curvature change imposed by the track installation machine. In time this short non-designed transition will evolve and lengthen even further as the trains are passing over it.

From this point of view we can say that **an actual sudden change in curvature is in fact impossible to install or maintain**, especially on ballasted track, because it will always tend to become a short curvature transition during installation respectively post-installation, due to the modelling effect of the passing trains.

Designing a sudden change in curvature

Because of the elements mentioned briefly here the transition curves are essential and necessary components of a good alignment design and their use should be generally part of a best practice design approach.
However there are locations where the transition curves are difficult to use within the limits of the design standards and also difficult to install on site, hence a sudden change in curvature can be considered and included in the design.
In order to limit and control these exceptions and ensure a still good, comfortable and safe train circulation, different railway administrations have developed two main approaches (see figure 3):

1. Limit the **virtual rate of change of cant deficiency, RcD (VT)**, calculated based on the assumptions of the *principle of Virtual Transition*.
   This is the design approach used in the UK and defined by the Track Design Handbook – NR/L2/TRK/2049 (2010) for Network Rail and by the track design standard S1157 (2014) for London Underground.
2. Limit the sudden change in curvature by limiting the **instantaneous change in cant deficiency (ΔD)**.
   This design approach is the most common used in continental Europe and around the world. It can be found in the European Norm for track alignment design parameters – BS EN 13803-2 (2006).

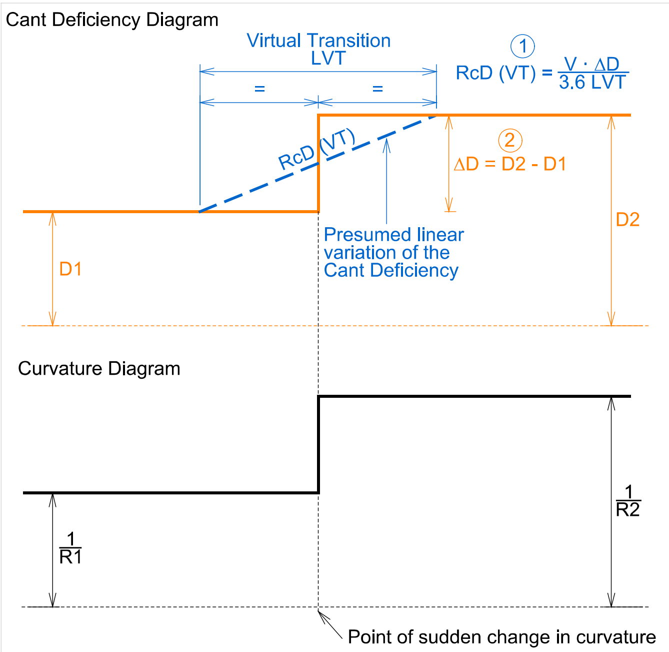


Figure 3. Curvature and Cant Deficiency Diagram in the area of a sudden change in horizontal curvature.

Principle of Virtual Transition

The Principle of Virtual Transition is defined in sheet B.3.4 of the Track Design Handbook – NR/L2/TRK/2049 (2010). A brief definition can be found in the informative Annex E of the European Norm for track alignment design BS EN 13803-2 (2006):
"This principle is based upon the assumption that a characteristic vehicle travelling over an abrupt change in curvature ***gains, or loses, cant deficiency (and/or cant excess) over a length equal to the distance between the bogie centres of the characteristic vehicle (B)****. This distance is also denoted virtual transition and is assumed to extend a distance B/2 each side of the abrupt change in curvature."*

The curvature variation over Virtual Transition

The principle of Virtual Transition is used only in areas where the cant is constant or zero. In this case the presumed variation of the Cant Deficiency will be caused only by the curvature variation perceived at vehicle level, when passing over a point of sudden change in curvature. Even though the standard does not specify how the cant deficiency varies over the Virtual Transition, in the alignment design this variation is considered linear, as for a real linear transition, the clothoid for example.
A detailed graphical description of the principle is given in figure 4. When the reference vehicle (Figure 4.A) is passing over the straight with both its bogies, the point M (the centroid of the vehicle) is staying on the straight track centreline. As the first bogie, C1, is passing over the tangent point TP to the circular curve, the point M is describing a smooth curve of unknown curvature. After the last bogie, C2, is passing the tangent point and both bogies are running on the circular curve, the point M describes an offset of this centreline circular curve (see Figure 4.B).

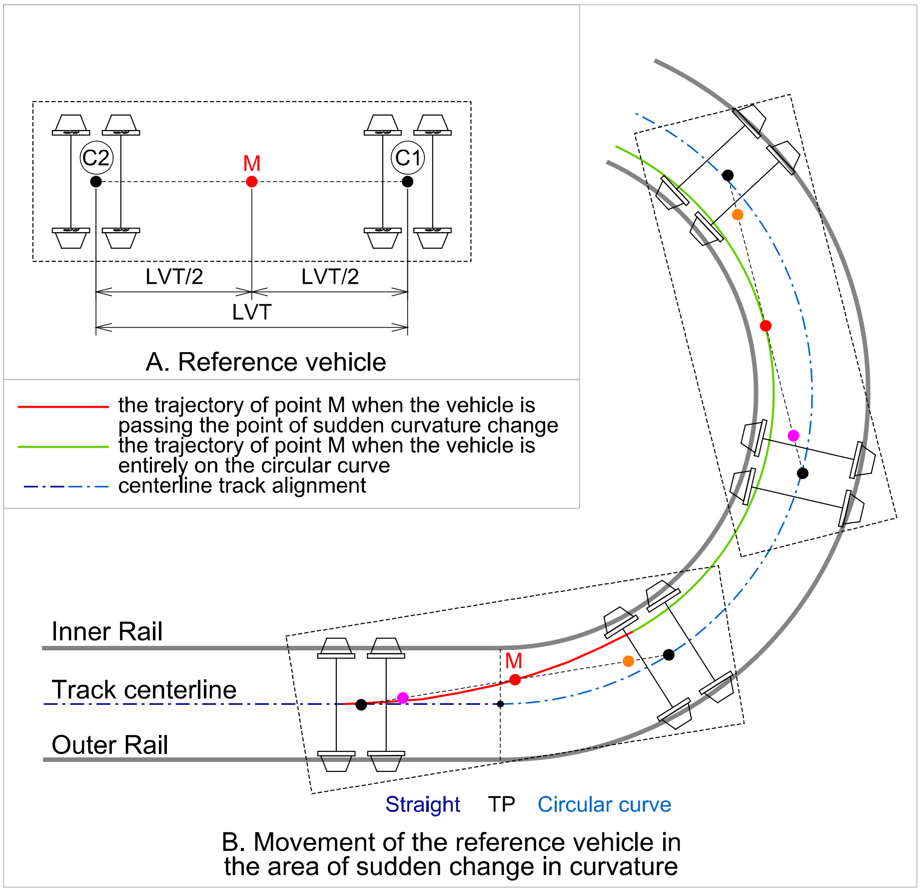


Figure 4. The trajectory of the middle point of the reference vehicle when passing a sudden change in curvature (not to scale)

Although the standard speaks about the movement of a vehicle, this is again considered in a simplified way, as a chord moving along the centerline of the track. The lateral movement of the bogies, the inclination of the vehicle towards the centre of the curve, the suspension dynamic and damping contribution are ignored when this design rule is defined. The effect of these ignored features is compensated by the limits imposed to the design parameters.
Again we a finding a simplified approach used in the standard to allow the definition of easy to understand and apply design rules. In the following paragraphs of this article, when referring to a vehicle we will consider the same simplification – the "vehicle" is considered the chord C1-C2, travelling with constant speed on the centerline of the track. The centre of mass of the vehicle is considered the middle point (M) of the chord C1-C2.

There are different ways to find out how the curvature varies over the virtual transition, either using acceptable approximate methods as the ones from the Hallade realignment theory (Radu – 2003), precise using spline polynomial equations, or even graphical methods, measuring the versines of the curve drawn by M as the vehicle is passing the tangent point.

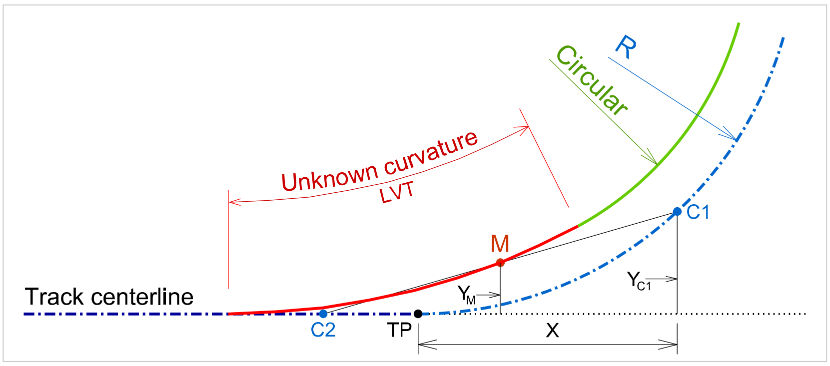


Figure 5. Finding the curve described by M (not to scale)

When the vehicle is passing the tangent point TP (see figure 5), the point C1 describes the centerline circular arc with the following equation:

$$Y_{C1} = R - \sqrt{R^2 - X^2} \quad (4)$$

$$= R - R\sqrt{1 - \frac{X^2}{R^2}}$$

Using Taylor expansion for the square root, Y_{C1} becomes:

$$Y_{C1} = R - R\left(1 - \frac{X^2}{2R^2} + \boxed{\frac{X^4}{8R^4} - \frac{X^6}{16R^6} + \cdots}\right) \quad (5)$$

Taking into account the large difference between X and R, the boxed part of this formula is practically zero. Hence, with acceptable precision, (5) becomes:

$$Y_{C1} = R - R\left(1 - \frac{X^2}{2R^2} + \mathbf{0}\right)$$

$$Y_{C1} = \frac{X^2}{2R} \quad (6)$$

For X significantly smaller than R, as it is the case for a railway track curve, this demonstrates the equivalence between the parabola, defined by (6), and a circle, defined by (4), with a radius R equal to the distance between the focus point and the directrix line of the parabola. This equivalence is known in analytical geometry as the osculating circle or "the kissing circle" of a parabola – *circulus osculans*.

Note: *The circle-parabola equivalence demonstrated here is used in railway and road design to define a "radius" for the standardised vertical parabola and to compute, for railway track, the tamper correction values.*
It can be also be used, within certain limits, as a justification for using circular vertical curves instead of vertical parabolas, recommended by the design and construction standard (see TRK2102, 6.7.5 Vertical Alignment).

Using the Thales theorem for point M, placed in the middle of the segment C1-C2, Y_M will be half of Y_{C1}:

$$Y_M = \frac{X^2}{4R} \quad (7)$$

Considering the proven equivalence relation between the equations (6) and (4) we can understand that the parabola defined by (7) is equivalent to a circle of radius Rm = 2R.
We can say with sufficient accuracy that, as the vehicle is passing the tangent point TP, the point M is describing a circle. The radius Rm of this circle is twice the radius R of the circular centreline curve.

In the same way it can be proven that the radius Rx of the circle described by any point X on the reference vehicle, located at the distance Lx from the trailing bogie centre C2, is:

$$Rx \cong R\frac{LVT}{Lx} \quad (8)$$

An easy, direct and more convincing way to prove this quasi-constant curvature is to check the evolution of versines in the areas of sudden change in alignment curvature (Figure 6). Converting the versines into the equivalent radius we will find with this graphical method similar results to the analytical one. When the reference vehicle (chord) is passing the tangent point TP, a point X, located at the distance Lx from the trailing bogie centre C2, is describing a circular arc of radius Rx. This radius will be found to be acceptable close to the one given by the formula (8).

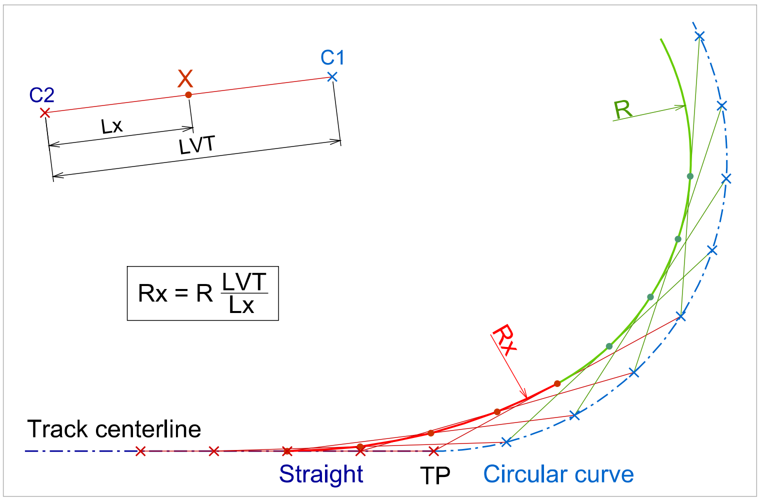


Figure 6. Graphical method to find out the Virtual Transition geometry (not to scale)

This exercise can be done for other radii and other alignment combinations. The results are similar – the intermediate constant curvature will always be found. This is true for the range of radii encountered in railway track design. The general form of this virtual curve is a polynomial spline curve, but for radii above 200m and chords (vehicles) of 12-20m the curvature is practically constant, as proven here.

The influence of the bogie was not taken into account but, using the same procedure, it can be proven that each bogie will add another short intermediate constant curvature. For point M, the middle point of the reference chord – assimilated to the vehicle centre of mass – this will bring the curvature variation acceptable close to the presumed linear curvature variation, justifying the definition of the virtual transition. As we move away from the centre of the vehicle the drop in curvature is more significant. For the points located at the bogie area the curvature variation is practically identical with the track curvature, showing the same sudden change in curvature, without any significant intermediate element.

Based on the equations presented here, it can be demonstrated that the gradual acquisition of angular velocity mentioned in the definition of the Virtual Transition Principle really happens. But the angular velocity and its variation is not considered a design constraint by the track design standards and none of the cant and alignment parameters, as they are defined today, are explicitly dependant of it. Nevertheless, its significance is highlighted briefly in the informative Annex A of the European Norm BS EN13803-1(2010).

This exercise demonstrates that the theory behind the Virtual Transition is more complex phenomenon, simplified by the standard to define an easy to understand and use design constraint, similar as concept with the design constraints for a real transition.

Kinematic modelling results

At speeds above 75mph the principle of virtual transition is starting to impose design constraints difficult to install on site, on ballasted track. Due to this reason, some discussions regarding the opportunity to change the 12.2m reference length for Virtual Transition, at higher speed, are taking place in the track design community.

The Track Design Handbook TRK2049 mentions that *"the value of 12.2m should not however be relaxed for vehicles with a longer wheelbase as the lateral thrust on track components becomes excessive."*
It seems logical to consider that the lateral thrust (or jerk) of the vehicle passing over a sudden change in curvature to increase linearly with speed. In strict mathematical terms this is true – but only if we still consider the simplifications commented previously in this article and ignore for example the suspension damping. A kinematic modelling of the vehicle can consider in an analytical way the very complex behaviour of different vehicle components. Dr Björn Kufver has presented such kinematic modelling results in several articles (Kufver and Heng – 2005). He concludes that "*the principle of virtual transition overestimates the lateral jerk for high speed trains and vehicles with short bogie distance. For a certain instantaneous change of cant deficiency, the principle of Virtual Transition presumes that the virtual rate of change of cant deficiency increases linearly with train speed. The vehicle dynamics simulations show no similar pattern for lateral jerk. In the simulations, the lateral jerk increases with speed, but converges to a certain maximum."*

Figure 7, reproduced from Kufver and Förstberg (2004), presents a set of simulation results for the Eurofima coach – the European Standard Coach, designed to be used on international railway routes in Europe (19m bogie centres). This shows the lateral jerk of the coach for different frequency filters when entering from straight to a curve with 100mm cant deficiency. Data series annotated "m" refer to the middle of the vehicle, while series annotated "w" refer to the worst position (above one of the bogies). The lateral jerk according to the principle of virtual transitions is also shown.

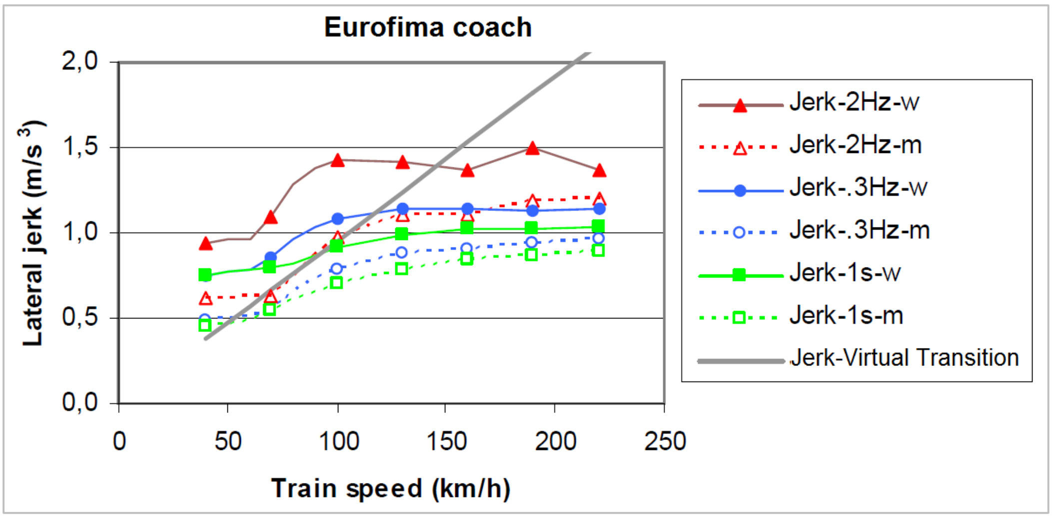


Figure 7. Lateral jerk for the Eurofima coach as a function of vehicle speed (Kufver and Förstberg 2004)

All these kinematic simulations have consistently shown the convergence of the lateral jerk above a certain speed. The lateral jerk at a speed of 100 km/h is about the same as for 220 km/h. Also, for this range of speeds, the lateral jerk has been proven to be significantly lower when compared to the one estimated based on the principle of virtual transition.

The limits of Rate of change in Cant Deficiency (RcD) – TRK2049

Using the principle of Virtual Transition, the Track Design Handbook TRK2049 is imposing for a sudden curvature change the same limits of the rate of change of Cant Deficiency as for a regular transition curve. These limits are defined in sheet B.2.3: Curving Design Values – Rate of Change of Cant Deficiency. For plain line and through route of S&C and permissible speed, these limits are quoted in Table 1.

Normal Design Value	Maximum Design Value	Exceptional Design Value
35 mm/s	55 mm/s	70 mm/s

Table 1. The limit of the Rate of Change of Cant Deficiency for plain line and through route of S&C (permissible speed)

For the turnout route of the S&C, sheet B.2.3 specify different limits of rate of change of cant deficiency, dependant on the turnout type. These are 55, 80 respectively 93.3mm/s (see TRK2049, sheet B.2.3). 93.3 mm/s applies only to the transitions in NR60 turnout routes.

Applying the principle of Virtual Transition, the designer computes the virtual rate of change of cant deficiency, RcD, for a 12.2 m virtual transition length.

$$RcD = \frac{\Delta D}{LVT} \cdot \frac{V\,[km/h]}{3.6} \quad (9)$$

Where LVT is the virtual transition length (12.2m) and ΔD is the sudden change in Cant Deficiency.
For constant cant, taking into account formula (2), this becomes:

$$RcD = 3.28 \cdot \frac{\Delta K \cdot \boldsymbol{V^3}}{LVT} \quad (10)$$

ΔK is the curvature variation:

$$\Delta K = \frac{1}{R_2} - \frac{1}{R_1} = \frac{R_1 - R_2}{R_1 R_2} \quad (11)$$

The limits of the instantaneous change in Cant Deficiency (ΔD) – BSEN 13803-2

For plain line alignments with sudden change in curvature the European Norm BSEN13803-2 states:
Plain line alignments with abrupt changes of cant deficiency shall only be used when the scope for designing the alignment is severely restricted. *Such restrictions occur in stations, at small deviations in alignment within a limited length, or in compound curves when there is only a small variation in the radii of abutting curves.*
The recommended values for abrupt change of Cant Deficiency on plain line shall be as specified in Table 2.

Speed V [km/h]	*V≤70*	*70<V≤170*	*170<V≤230*
Recommended values ΔD_{lim} [mm]	*50*	*40*	*30*

Table 2. Recommended values of abrupt change in Cant Deficiency (ΔD_{lim}), according to BSEN 13803-2

For S&C layouts the limits for conventional lines are presented in table 3:

Speed V [km/h]	V≤100	100<V≤170	170<V≤220	220<V≤230
Recommended values ΔD_{lim} [mm]	100	133-0.33V		60
Maximum limiting values ΔD_{lim} [mm]	120	141-0.21V	161-0.33V	

Table 3. Limiting Values of abrupt change of cant deficiency (ΔD_{lim}) – Conventional lines, according to BSEN 13803-2

Note: A tolerance of 10mm on the maximum limiting values is permitted for existing turnouts laid on lines to be upgraded to high-speed

Comparison between TRK2049 and BS EN 13803-2 for the limits of sudden change in Cant Deficiency

In order to compare these two design approaches we need to compute the equivalent sudden change in cant deficiency, ΔD, for the limits defined in the Track Design Handbook, NR/L2/TRK/2049.

A graphical comparison between the two standards is presented in figure 8.

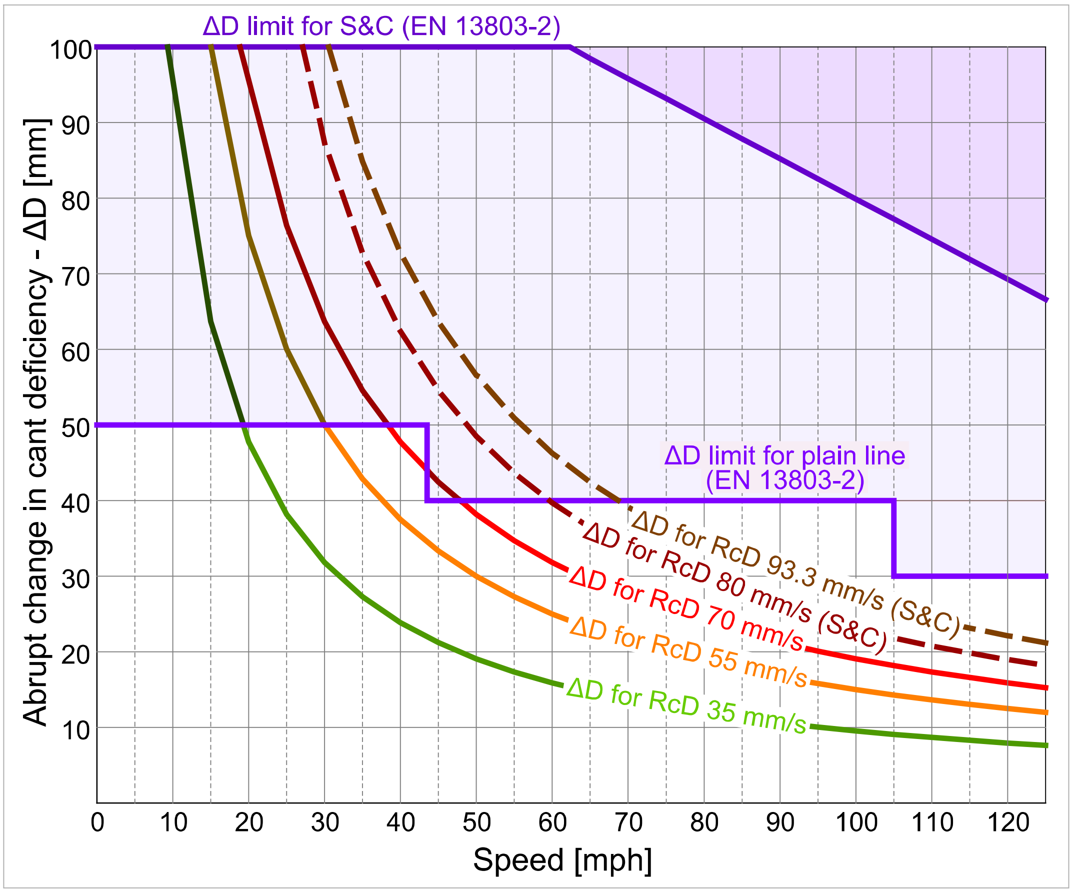


Figure 8. Comparison between the design restrictions for a sudden change in curvature (ΔD was computed from RcD for a virtual transition length LVT of 12.2m)

Although it has constant limiting values, *it must be noted that ΔD is also considering the speed, as it is demonstrated at the beginning of this article. The cant deficiency, even though is "measured" in millimetres is in fact the expression of the non-compensated lateral acceleration, dependant on speed and curvature. By stating constant limits for the sudden change in cant deficiency, the European Norm is in fact imposing constant limits for the sudden variation of the non-compensated lateral acceleration.*

Figure 8 shows that, at low speed, the sudden change in cant deficiency limited by TRK2049 is close or even above the limits from BSEN 13803-2, for plain line. In this area, of low speeds, it is justified to use the best practice rule to limit virtual rates of change of cant deficiency to 20mm/s (Barnard, 2013).
In order to place the European Norm rules in the UK design perspective we can apply the principle of Virtual Transition and compute the virtual rate of change in cant deficiency for different speeds and the ΔD limits shown in figure 8 for plain line and S&C. These virtual rates of change in cant deficiency is shown in figure 9.

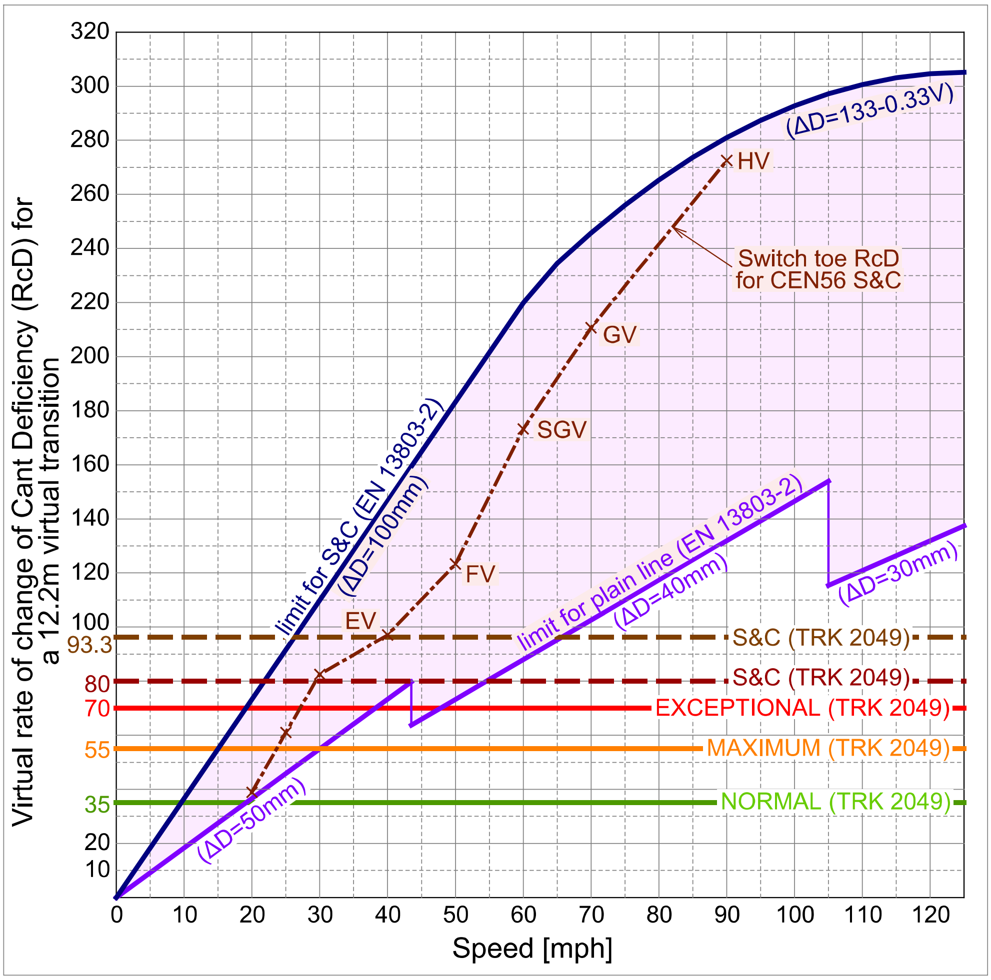


Figure 9. Comparison between the design restrictions for a sudden change in curvature (the equivalent virtual RcD for EN13803 is computed for a virtual transition length LVT of 12.2m)

The graph shows very high virtual rates of change of cant deficiency of the European Norm, well above the ones found in TRK2049.

It must be noted here that this difference between the two standards is specifically for a sudden change in curvature when cant is constant or zero, when the cant is kept constant and where "the alignment is severely restricted" (BS EN13803-2, 2006). Where cant variation is involved the European Norm EN13803-1 defines very similar design constraints to the British standards.

The high values of the virtual rate of change of cant deficiency shown by the European Norm are nevertheless similar with the ones found at the switch toes of some of the NR standardised S&C. Here TRK2049 allows the rate of change of cant deficiency to be disregarded (See TRK2049, B.2.3 sheet 57). Figure 9 includes these rates of change of cant deficiency, computed for the planning radius of CEN 56E1 Vertical S&C.

The limits of sudden change in Cant Deficiency according to the German track alignment standard

The limits for the sudden change in Cat deficiency (ΔD) defined in EN 13803-2 and presented here are found for a long time in most of the national design standards across Europe; one significant example is RIL 800.0110 – the German track alignment design standard. The use of these limits for sudden change in curvature has not caused increased wear, or any other significant negative effect that might require a change in the national design rules or to highlight this as a risk during the development and implementation of the European Norm EN 13803-2.

The 2008 version of the German standard RIL 800.0110 gives minimum radii not requiring transition curves. Table 4 replicates the German minimum radii table, presenting also the sudden change in cant and the virtual rate of change of Cant Deficiency, computed for a 12.2m virtual transition.

Speed [mph]	RIL 800.0110 specifications			Sudden change of Cant Deficiency ΔD		Equivalent virtual Rate of change of Cant Deficiency for 12.2m virtual transition **RcD** [mm/s]	
	Speed [km/h]	Minimum radius not requiring transition curve [m]					
		Plain line	S&C	Plain line	S&C	Plain line	S&C
25	40	220	180	86	105	**79**	**96**
32	50	340	280	87	106	**100**	**121**
38	60	490	400	87	107	**119**	**147**
44	70	670	545	87	107	**139**	**171**
50	80	875	710	87	107	**159**	**195**
56	90	1110	900	87	107	**179**	**220**
63	100	1370	1110	87	107	**199**	**244**
69	110	1735	1410	83	102	**208**	**256**
75	120	2170	1745	79	98	**216**	**268**
81	130	2680	2130	75	94	**222**	**279**
87	140	3275	2575	71	90	**227**	**287**
94	150	3990	3085	67	87	**229**	**298**
100	160	4825	3675	63	83	**230**	**303**
106	170	5810	4350	59	79	**229**	**306**
112	180	6975	5125	55	75	**226**	**308**
119	190	8365	6000	51	71	**221**	**308**
125	200	10000	7000	48	68	**219**	**310**

Table 4. Cant deficiency parameters for the minimum radius not requiring transition to straight according to the German track alignment design standard RIL 800.0110 (2008)

Comparing the European and German limits with the ones found for Virtual Transition in the Track Design Handbook, we can notice that the British standard is more conservative and potentially imposing too safe limits for sudden curvature change at high speed.

Again, the German S&C minimum radii presented here are similar with the switch radii used in UK, hence the rates of change of cant deficiency will be similar. For example, for 90mph the switch toe rate of change in cant deficiency of a German S&C is 290mm/s and for a HV – CEN56E1 Vertical S&C is 273mm/s.

Lost in small numbers…

The large numbers shown till now need to be considered also in the site installation context. The curve shift is a subtle but important parameter showing how significant the installation of a transition curve really is. When a transition is to be installed between two circular curves (see figure 10) one of the curves is shifted towards the centre compared to the other.

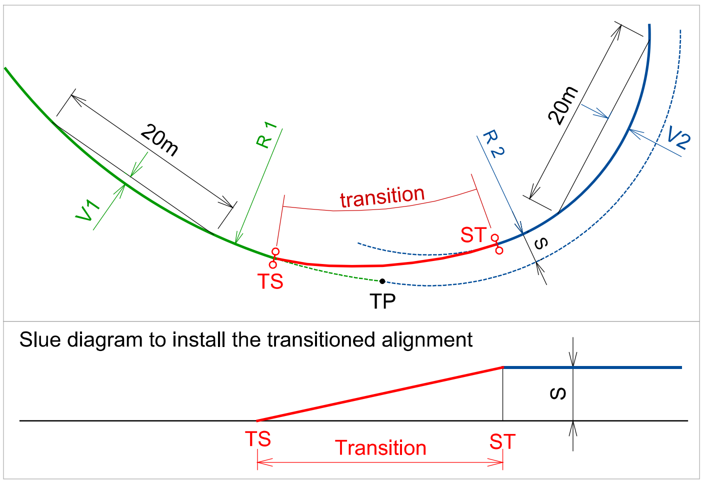


Figure 10. Curve shift S to allow the installation of a transition curve

This shift (theoretical slue), S, for a clothoid transition, is dependent on the curvature variation ΔK (11) between the two curves:

$$S = \frac{L^2}{24}\Delta K - \frac{L^4}{2668}\Delta K^3 + \cdots \qquad (12)$$

In some European countries the following best practice rule for the specification of a transition curve is applied: *if the required curve shift to install a transition is below 3mm, that transition should not be proposed in the design as it is practically impossible to be installed on site, on ballasted track.*
Table 5 presents the curve shift required for a 30m transition, when the virtual rate of change of cant deficiency for a virtual transition is 1mm/s above the limits defined by the track design standard. It shows also the shift for the case when the sudden change in cant deficiency ΔD is 1mm above the limit defined by the European Norm EN13803-2.
The table includes the versine variation between R1 and R2 for each specific design constraint.

Speed		Virtual RcD (mm/s) TRK2049 Limit +1mm/s			EN 13803-2 ΔD+1mm	Virtual RcD (mm/s) TRK2049 Limit +1mm/s			EN 13803-2 ΔD+1mm
V [mph]	**V [km/h]**	36 (35+1)	56 (55+1)	71 (70+1)		36 (35+1)	56 (55+1)	71 (70+1)	
		Shift required to insert a 30m transition (mm)				**Versine variation measured for a 20m chord (mm)**			
60	96.6	5.6	8.7	11	14	7.5	11.6	14.7	18.7
65	104.6	4.4	6.9	8.7	11.9	5.9	9.1	11.6	15.9
70	112.7	3.6	5.5	7	10.3	4.7	7.3	9.3	13.7
75	120.7	2.9	4.5	5.7	9	3.9	6	7.6	12
80	128.7	2.4	3.7	4.7	7.9	3.2	4.9	6.2	10.5
85	136.8	2	3.1	3.9	7	2.7	4.1	5.2	9.3
90	144.8	1.7	2.6	3.3	6.3	2.3	3.5	4.4	8.3
95	152.9	1.5	2.2	2.8	5.6	1.9	3	3.7	7.5
100	160.9	**1.3**	1.9	2.4	5.1	**1.7**	2.5	3.2	6.7
105	169.0	1.1	1.7	2.1	4.6	1.4	2.2	2.8	6.1
110	177.0	1	1.5	1.8	3.2	1.3	1.9	2.4	4.2
115	185.1	0.8	1.3	1.6	2.9	1.1	1.7	2.1	3.9
120	193.1	0.7	1.1	1.4	2.7	1	1.5	1.9	3.6
125	201.2	**0.7**	1	1.3	2.5	**0.9**	1.3	1.7	3.3

Table 5. Circular curve shift and versine variation for different design limits

By inserting a 30m transition between R1 and R2 (hence shifting R2 by the values presented in table 5), the rate of change of cant deficiency changes as follows:

- From 36mm/s to 15mm/s (**21mm/s** decrease)
- From 56mm/s to 23mm/s (**33mm/s** decrease)
- From 71mm/s to 29mm/s (**42mm/s** decrease).

For example, according to the current design rules, for 100mph the design proposal should include a transition curve if the virtual rate of change of cant deficiency RcD, computed for a 12.2m virtual transition, is 36mm/s (1mm/s above the Normal design limit). The minimum length of this transition is 30m. In order to install this transition, the curve R2 needs to be slued **1.3mm** compared to the alignment without transition. This transition installation and the **1.3mm** shift of the curve, will provide then a 15mm/s rate of change of cant deficiency (**21mm/s** decrease).
This case will appear for example if R1 = 1200m and R2 = 1250m. For both curves the cant is 150mm. The versines for these curves, measured for a 20m chord, are v1 = 41.7mm and v2 = 40mm and the versine variation is **1.7mm**.
In other words, if the cant is constant or zero, at 100mph the insertion of a 30m transition by slueing the second circular curve **1.3mm** is causing a **21mm/s** decrease in the rate of change of cant deficiency.
In fact, dependant on the local alignment constraints, often the shift is not constant throughout the R2 arc but reaches a maximum value near the spiral end point ST and decreases as we move away from it. Hence, the real slue values mentioned above might be even lower.

It seems that in the range of higher speed the designer is “trapped” by the virtual transition standard rule in a theoretical game of very small numbers. The shifts and versine variations commented here seem more theoretical values rather than figures that can be installed properly on site, within the accepted geometrical track tolerances defined by NR/L2/TRK/2102 (2010) – Table A.1 (ballasted track) or A.2 (slab track).

Table 5 gives also an indication on when a slab track structure should be used, as it will ensure a more precise installation of the desired geometry. The slab track is not as flexible laterally as the ballasted track hence more careful consideration should be given when defining the track alignment geometry, especially the type and variation length for transition and cant.

The entire discussion presented in this article proves that, as the speed increases, some design parameters, important at normal speed, lose or change their significance as the speed increases. On the other hand, other, more subtle design parameters - ignored in our current design approach - might become significant. Hence simple extrapolation of the current design rules cannot always be applied. In this specific case the cubed speed in formula (10) is causing, at higher values, the very tight constraints discussed here.

The transition curves are essential and necessary elements of a good alignment design and their use should always be part of a best practice design approach.
However there are locations where the transition curves are difficult to be used within the limits of the design standards and also difficult to install on site, hence a sudden change in curvature is needed to be included in the design.
The purpose of this article was to compare different approaches on how this sudden change in curvature is designed and perhaps to facilitate further discussion on this subject. It is not intended to be an alternative design guide but just an informative paper.

Acknowledgements

The author thanks the following for their comments, suggestions and support in writing this article:
Constantin Radu, *Univ Prof Dr Eng – TUCE Bucharest,*
Geoffrey Kennedy, *Professional Head of Discipline – Permanent Way, Atkins,*
Andy Raglan, T*echnical Authority: Track Design– Permanent Way, Atkins,*
Levente Nogy, *Senior Design Engineer, Network Rail,*
Graham Barnard, *Chief Engineer – Track, Amey,*
Geoff Bean, *Project Engineering Manager, Network Rail,*
Steven Nuttall, *Principal Permanent Way Engineer, Atkins*.

References

- BS EN 13803-1 (2010) *Railway Applications – Track – Track alignment parameters – Track gauges 1435 and wider – Part 1: Plain Line.* British Standards Institution, London.
- BS EN 13803-2 (2006) *Railway Applications – Track – Track alignment parameters – Track gauges 1435 and wider – Part 2: Switches and crossings and comparable alignment design situations with abrupt changes of curvature.* British Standards Institution, London.
- GC/RT5021 (2011) *Track System Requirements*, Issue Five, Rail Safety and Standard Board Limited, London.
- NR/L2/TRK/2049 (2010) *Track Design Handbook,* Network Rail.
- NR/L2/TRK/2102 (2010) *Design and Construction of Track,* Network Rail.
- RIL 800.0110 (1999 / 2008) *Richtlinie - Netzinfrastruktur Technik entwerfen – Linienführung (Directive – Infrastructure Network Technical design – Lines),* Deutsche Bahn. (German track alignment design standard)
- S1157 (2014) *Track – Performance, Design and Construction*. London Underground.
- Barnard, Graham (2013) *Virtual Transitions*. The Journal, Vol 131 – part 3. Permanent Way Institution.
- Cope, Geoffrey (1993) *British Railway Track – Design, Construction and Maintenance*, Permanent Way Institution, Echo Press, Loughborough.

- Kufver, Björn; Förstberg, Johan (2004). *Dynamic vehicle response versus virtual transitions*. Proceedings of Comprail 2004, 17-19 May 2004, Dresden. WIT Press, Southampton.
- Kufver, Björn; Heng, Boon Seng (2005) *Kinematics on virtual transitions*. Paper presented at the conference: RAILWAY ENGINEERING - 2005, University of Westminster, London.
- Network Rail (2008) *Improving Alignment Designs*, I&I Track Programme, Network Rail.
- Radu, Constantin (2003) *Rectificarea si retrasarea curbelor de cale ferata (Rectification and realignment of the railway track curves)*. Track geometry course. Technical University of Civil Engineering Bucharest.
- Radu, Constantin; Ciobanu, Constantin (2004) *Elemente referitoare la utilizarea parabolei cubice imbunatatite (Elements related to the use of the improved cubic parabola)*, Third Romanian National Railway Symposium, Technical University of Civil Engineering Bucharest.
- Plasser&Theurer (2015) *Plasser&Theurer history* – [online article] Available from: http://www.plassertheurer.com/en/company/history.html [Accessed: 26th May 2015]